FAR BEYOND

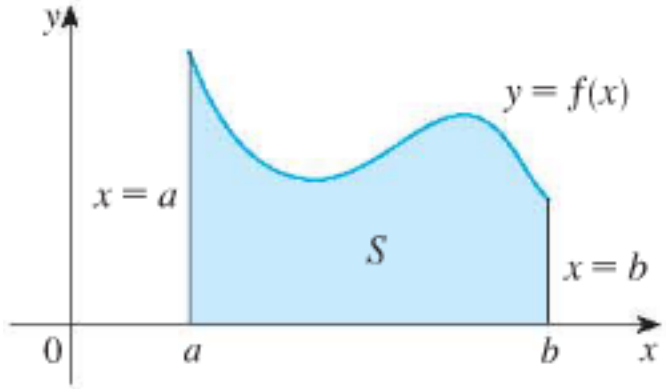
MAT122

Area Under Standard Shapes

Stony Brook University

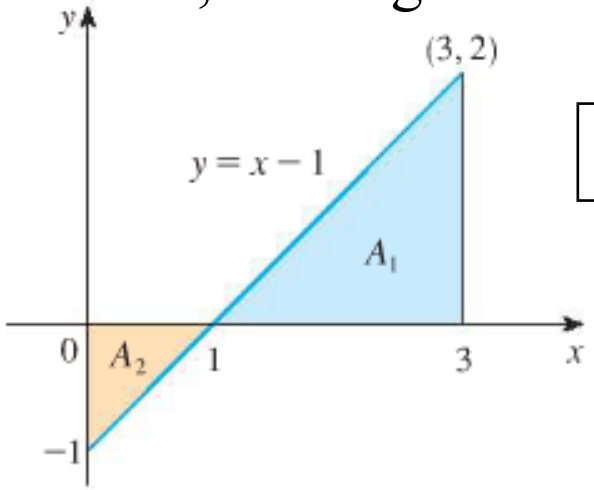
Area Under a Curve - Intro

It becomes a challenge to determine areas of non-standard shapes such as the bound area under a function curve.



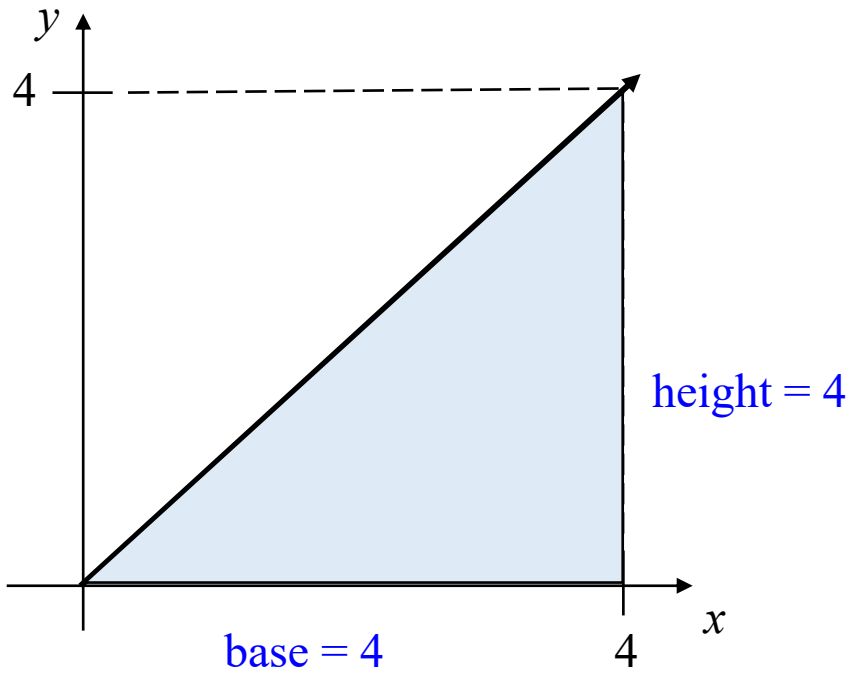
If area under curve is a <u>standard</u> shape, e.g., rectangle, triangle, circle, trapezoid then known area formulas can be used.

Caveat: If area lies **below** x-axis, that region will be <u>negative</u> (subtracted).

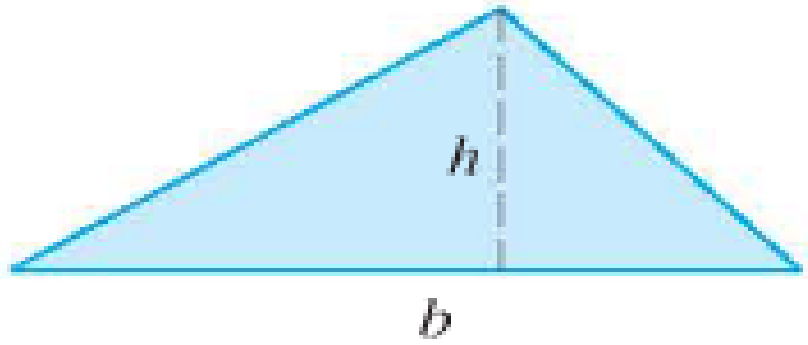


Total area of shaded regions: $A_1 - A_2$

Area Under a Line - Triangle



Area of a Triangle:



$$A = \frac{1}{2}bh$$

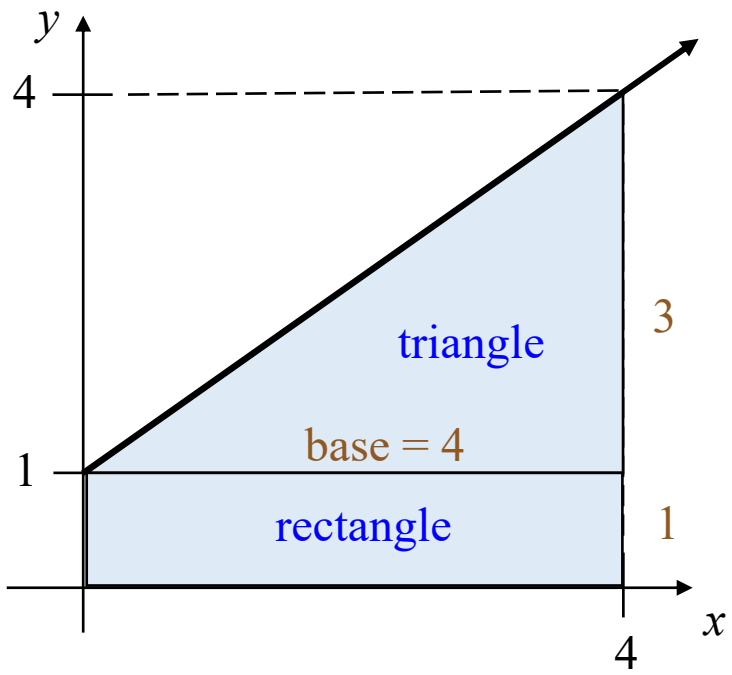
ex. Find the area under the line from $x = 0$ to $x = 4$.

$$A = \frac{1}{2} \cdot b \cdot h$$

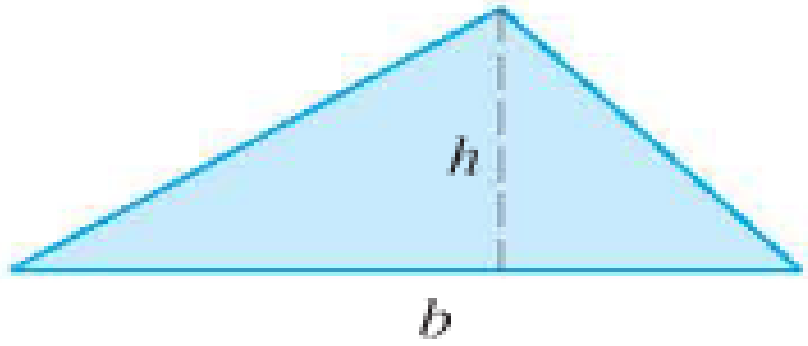
$$= \frac{1}{2} \cdot 4 \cdot 4$$

$$= \frac{1}{2} \cdot 16 = \boxed{8}$$

Area Under a Line – Triangle, Rectangle

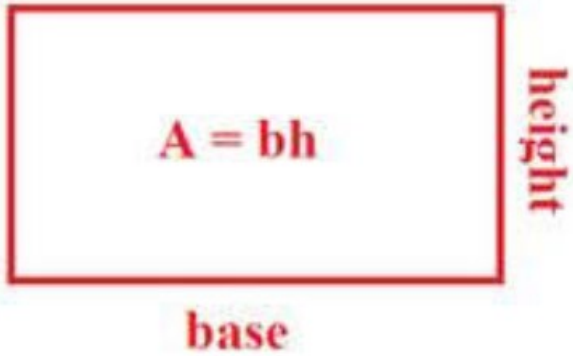


Area of a Triangle:



$$A = \frac{1}{2}bh$$

Area of a Rectangle:



ex. Find the area under the line from $x = 0$ to $x = 4$.

Area under line $= A_{triangle} + A_{rectangle}$

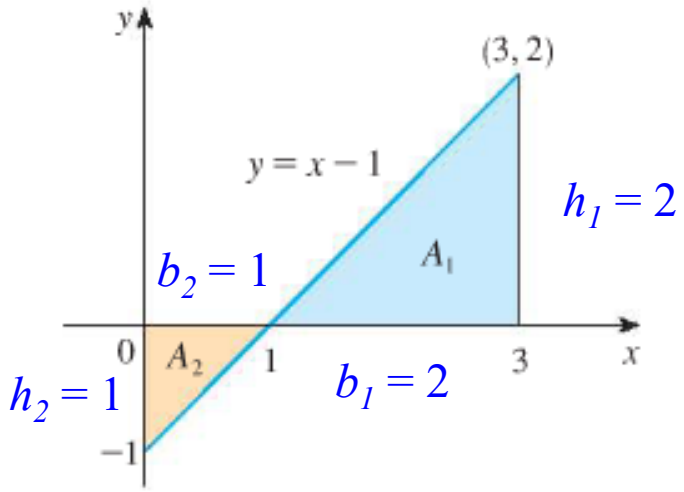
$$= \frac{1}{2} \cdot b_1 \cdot h_1 + b_2 \cdot h_2$$

$$= \frac{1}{2} \cdot 4 \cdot 3 + 4 \cdot 1$$

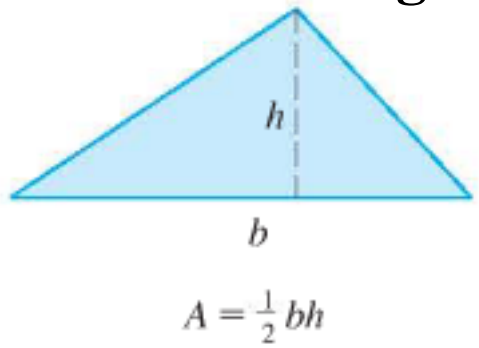
$$= 6 + 4 = \boxed{10}$$

Area Under a Line – with negative

ex. Calculate shaded area under $y = x - 1$ between $x = 0$ and $x = 3$.



Area of a Triangle:



$$A_1 = \frac{1}{2} b_1 h_1 = \frac{1}{2}(2)(2) = 2$$

$$A_2 = \frac{1}{2} b_2 h_2 = \frac{1}{2}(1)(1) = \frac{1}{2}$$

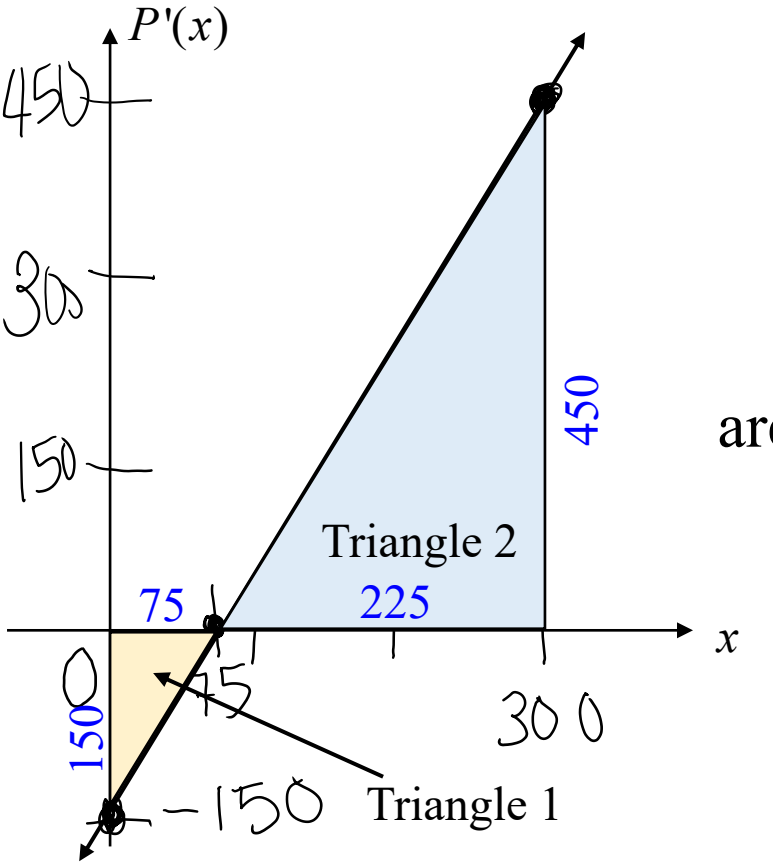
$$A_1 - A_2 = 2 - \frac{1}{2} = \boxed{1\frac{1}{2}}$$

Total area of shaded regions: $A_1 - A_2$

Integration Application

ex. A concert promoter sells x tickets and has a marginal-profit given by $P'(x) = 2x - 150$, where $P'(x)$ is in dollars/ticket. Find the total profit, $P(x)$, from the sale of the first 300 tickets.

Technique: find exact area under curve from $x = 0$ to $x = 300$



get x-intercept: $0 = 2x - 150$

$2x = 150$

$x = 75$

get y-intercept: $(0, -150)$

$P'(300) = 2(300) - 150$

$= 600 - 150$

$= 450$

area under "curve" $=$ $\text{Area}_{\text{Triangle 2}} - \text{Area}_{\text{Triangle 1}}$

$= \frac{1}{2} \cdot 225 \cdot 450 - \frac{1}{2} \cdot 75 \cdot 150$

$= 225 \cdot 225 - 75 \cdot 75$

$= 50{,}625 - 5625 = \boxed{\$45{,}000}$

$$A_{\Delta} = \frac{1}{2}bh$$